- $z = r(\cos\theta + i\sin\theta)$ **(Polar Form)**
- **Euler's Form** $\mathbf{z = re^{i\theta}}$ where $e^{i\theta} = \cos\theta + i\sin\theta$
- $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$ provided $z_2 \neq 0$
- $z = \bar{z}$ iff z is purely real
- $z = -\bar{z}$ iff z is purely imaginary.
- $|z_1 z_2| = |z_1|\,|z_2|$
- $|z^n| = |z|^n$
- $\mathbf{|z\bar{z}| = |z|^2}$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\ (z_2 \neq 0)$
- $|z_1 + z_2| \leq |z_1| + |z_2|$
- $|z_1 - z_2| \leq |z_1| + |z_2|$ and $|z_1 - z_2| \geq \big||z_1| - |z_2|\big|$

  i.e. $\mathbf{\big||z_1| - |z_2|\big|} \leq |\mathbf{z_1 - z_2}| \leq |\mathbf{z_1}| + |\mathbf{z_2}|$
- $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\,\mathrm{Re}\left(z_1\bar{z}_2\right) = |z_1|^2 + |z_2|^2 \pm \left(z_1\bar{z}_2 + \bar{z}_1 z_2\right)$
- $\arg(z_1 . z_2) = \arg(z_1) + \arg(z_2)$
- $\arg(z^n) = n.(\arg(z))$
- $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
- If arg(z) = 0 or $\pi$ then z will be a **purely real no.**
- If arg(z) = $\frac{\pi}{2}$ or $-\frac{\pi}{2}$ then z will be puerly Imaginary no.

**de Moivre's theorem:**

- $\mathbf{(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta}$ **for n being an integer** and $(\cos\theta - i\sin\theta)^n = \cos n\theta - i\sin n\theta$,

- **Cubic Roots of unity**

Let $x = 1^{\frac{1}{3}}$ $\Rightarrow x = (\cos 0 + i\sin 0)^{\frac{1}{3}} = (\cos 2k\pi + i\sin 2k\pi)^{\frac{1}{3}}$

$\Rightarrow x = \cos\left(\frac{2k\pi}{3}\right) + i\sin\left(\frac{2k\pi}{3}\right)$, where k = 0, 1, 2

For k = 0, x = cos 0 + i sin 0 = **1**

For k = 1, $x = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = \frac{-1}{2} + \frac{i\sqrt{3}}{2} = e^{i\frac{2\pi}{3}} = \omega$

For k = 2, $x = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = \frac{-1}{2} - \frac{i\sqrt{3}}{2} = e^{i\frac{4\pi}{3}} = \omega^2$

**Cube roots of unity** $= \mathbf{1, \omega, \omega^2}$

- **Properties of cube roots of unity:**

(i) Sum of the roots $1 + \omega + \omega^2 = 0$

(ii) Product of the roots $\omega^3 = 1$

(iii) Roots are in GP.

(iv) Lie on a unit circle $|z| = 1$ or $x^2 + y^2 = 1$

## Geometry of Complex Numbers in Argand Plane:

### Basics relating complex number and coordinate geometry :

- $z$ (= $x+iy$ ) represents variable point P (x, y) in argand plane.
- $z_1$ ( = $x_1+iy_1$) represents fixed point A ($x_1$, $y_1$) in argand plane.
- $|z_2 - z_1|$ represents length of line segment AB , where A is "$z_1$" and B is "$z_2$".
- $z_2 - z_1$ can be thoght of as a vector ( directed line segment ) $\vec{AB}$ , where A is "$z_1$" and B is "$z_2$".
- arg ($z_2 - z_1$) is angle made by line segment AB from direction of x-axis ( real axis ) of argand plane.

**Locus / Shapes in argand plane :**

- If C be the point dividing line segment joining A and B in the ratio m : n internally; Point C in argand plane will be represented as $\frac{mz_2 + nz_1}{m+n}$.

**$|z_1 - z_2|$ and associated locus:**

- **If $|z_2 - z_1| = 5$ ;** this means that **length of line segment AB** is equal to 5 units ,( please note point A is represented by $z_1$ and point B is by $z_2$ in argand plane. )
- **If $|z| = 5$** or $|z-0| = 5$; this means that variable ( moving point ) point **P( z) is always at "5" unit distance from origin O** (0,0) .

  i.e. point P is on the circumference of circle whose centre is (0,0) and radius = 5 units.

  In other words , for $|z| = 5$ , locus represented by moving point "z" is a circle with centre at (0,0) and radius = "5" units.
- **$|z - 4| = 3$ ;** locus of "z" is a circle with centre @ ( 4,0) and radius = 3 units in argand plane.

- **$|z-(4-3i)| = 5$ ;** locus of "z" is a circle with centre @ ( 4, –3) and radius = 5 units. It can also be contrued as "collection all such points (z) which are **on the** circumference of a circle with centre @ ( 4, –3) and radius = 5 units"

- **If $|z-(1+2i)| \geq 4$ ;** collection of all the point either **on or outside** the circle with centre @ ( 1, 2) and radius = 4 units.

- **For $|z-(1+2i)| \leq 4$ ;** collection of all the point either **on or inside** the circle with centre @ ( 1, 2) and radius = 4 units.

- **If $|z-z_1| = |z-z_2|$ ;** then moving point P is such that length of line segment AP = BP ; where A is represented by $z_1$ and point B is by $z_2$.

  As AP = BP ; locus of **P is perpendicular bisector of line segment AB**

- If $|z-z_1|+|z-z_2| = |z_1-z_2|$ ; point P(z) is on the **line segment** AB

- **If $|z-z_1| + |z-z_2| = k\ (>|z_1-z_2|)$** ; moving point P(z) is such that AP + BP = k ( > AB) .

  Hence , P lies on the **ellipse** whose focii are A and B , with A nd B being fci of the ellipse and "k" is equal to the length of major axis

- If $|z-z_1|-|z-z_2| = K\left(<|z_1-z_2|\right)$ ; moving point P(z) is such that AP – BP = k ( > AB) .

  Hence , P lies on the **hyperbola** whose focii are A and B , with A nd B being fci of the ellipse and "k" is equal to the length of transverse axis

- If $|z-z_1|^2+|z-z_2|^2 = k$ (constant) ; then locus of "z" will be a circle if $k \geq \frac{1}{2}|z_1-z_2|^2$ .

- If $|z-z_1| = k\,|z-z_2|$ (Where k > 0 and ≠ 1) ; then "z" lies on a **circle and if k = 1 ;** z will lie on perpendicular bisector of line segment AB.

**arg ($z_1$–$z_2$) and associated locus**

- **If arg($z_1$) = $\frac{\pi}{6}$** or **arg($z_1 - 0$) = $\frac{\pi}{6}$** ; it means that OA is making an **angle equal to $\frac{\pi}{6}$ f**rom positive direction of x- axis ( real axis).

- **If arg (z) = $\frac{\pi}{6}$ or arg (z–0) = $\frac{\pi}{6}$ ;** it means that mpving point P (z) will **on the ray** emanating from origin at an angle of $\frac{\pi}{6}$ from positive direction of x- axis ( real axis).

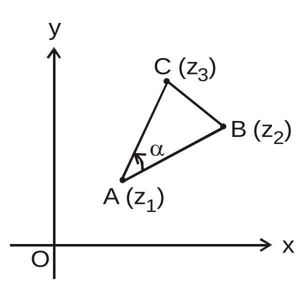
- If arg ($z_2 - z_1$) = $\frac{\pi}{4}$ ; it means that directed line segment AB is **making $\frac{\pi}{4}$ from horizontal direction** ( parallel to x - axis) ; where A is represented by "$z_1$" and B is represented by "$z_2$" .

- If $\arg\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{3}$ or $\arg(z - z_1) - \arg(z - z_2) = \frac{\pi}{3}$ ;

  it means that angle between line segments AP and BP is $\frac{\pi}{3}$ ; when angle is measured with arrow poniting towards AP and where A is represented by "$z_1$" and B is represented by "$z_2$" .
  Hence, moving point P(z) will be on the major arc of the circle with chord being AB.

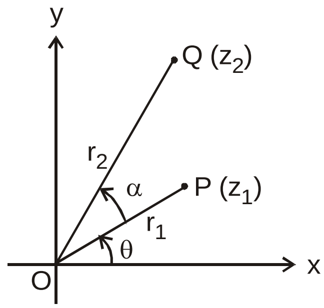
- If $\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha$ ( $\neq 0, \pi, -\pi$) then locus of z is an arc of circle with chord being AB; where A is represented by "$z_1$" and B is represented by "$z_2$" .

- If $z_1$, $z_2$, $z_3$ represent points A, B, C in the Argand plane, then $\angle BAC = \alpha = \arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)$



## Multiplication of complex numbers associated to Rotation of line segments

- If $z_1 = r_1 e^{i\theta}$ and $z_2 = r_2\, e^{i(\theta+\alpha)}$ ; where $r_1 = |z_1|$ and $r_2 = |z_2|$




$\therefore\ z_2 = r_2 e^{i(\theta+\alpha)} = \frac{r_2}{r_1} r_1 e^{i\theta}.e^{i\alpha} = \left|\frac{z_2}{z_1}\right| z_1 e^{i\alpha}$ or $\mathbf{\frac{z_2}{|z_2|} = \frac{z_1}{|z_1|}.e^{i\alpha}}$ ( analogous to Unit vector concept)

Hence, $\mathbf{z_2 = \left(\frac{|z_2|}{|z_1|}\right)\left(e^{i\alpha}\right).z_1}$ ; i.e. to obtain $z_2$ from $z_1$ , we need two transformations ;

(1) Modulus Transformation : $\left(\frac{|z_2|}{|z_1|}\right)$

(2) Angular Transformationa : $\left(e^{i\alpha}\right)$

- If A $(z_1)$ , B$(z_2)$ and C$(z_3)$ are vertices of an Equilateral Triangle ABC

then $\mathbf{z_1^2 + z_2^2 + z_3^2 - z_1z_2 - z_2z_3 - z_3z_1 = 0.}$

- Proof : as we know $|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$ (AB = BC = CA) and

$z_1 - z_2 = (z_3 - z_2).e^{\frac{i\pi}{3}}$ ; $z_2 - z_3 = (z_1 - z_3)e^{\frac{i\pi}{3}}$ $\therefore e^{i\frac{\pi}{3}} = \left(\frac{z_1 - z_2}{z_3 - z_2}\right) = \left(\frac{z_2 - z_3}{z_1 - z_3}\right)$

By solving , we get $\mathbf{z_1^2 + z_2^2 + z_3^2 - z_1z_2 - z_2z_3 - z_3z_1 = 0.}$

**1.** The argument of $\frac{z_1}{z_2}$ if $z_1 = \sqrt{3} + 3i$ and $z_2 = 1 - i$ is

(1) $-\frac{5\pi}{12}$ (2) $\frac{7\pi}{12}$

(3) $\frac{\pi}{12}$ (4) $-\frac{\pi}{3}$

**2.** The value of arg (x) , when x < 0 :

(1) 0 (2) $\frac{\pi}{2}$

(3) $\pi$ (4) None of these

**3.** If $z = \left(\sqrt{3}+i\right)^7 + \left(\sqrt{3}-i\right)^7$ then

(1) Re (z) = 0 (2) Im (z) = 0

(3) Re (z) > 0, Im (z) > 0 (4) Re (z) > 0, Im (z) < 0

**4.** The value of $(\sin\theta + i\cos\theta)^5$ is

(1) $\sin 5\theta + i\cos 5\theta$ (2) $\sin 5\theta - i\cos 5\theta$

(3) $\cos 5\theta + i\sin 5\theta$ (4) $\cos 5\theta - i\sin 5\theta$

**5.** If $\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{2015}$ = a + ib, then (a, b) =

(1) $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ (2) $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

(3) $\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$ (4) $\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$

**6.** What is the value of $i^i$?

(1) $\infty$ (2) i

(3) $e^{-\frac{\pi}{2}}$ (4) Not defined

**7.** If $(x_1 + iy_1)(x_2 + iy_2) \ldots (x_n + iy_n) = A + iB$, then $\sum_{i=1}^{n}\tan^{-1}\left(\frac{y_i}{x_i}\right) =$

(1) $\frac{B}{A}$ (2) $\tan\left(\frac{B}{A}\right)$

(3) $\tan^{-1}\left(\frac{B}{A}\right)$ (4) $\tan^{-1}\left(\frac{A}{B}\right)$

**8.** Let $z_1$ and $z_2$ be two complex numbers such that $\overline{z_1} + i\overline{z_2} = 0$ and $\arg(z_1 z_2) = \pi$, then $\arg(z_1)$

(1) $\pi$ (2) $\pi/2$
(3) $2\pi/3$ (4) $3\pi/4$

**9.** If $z_1$, $z_2$ and $z_3$, $z_4$ are two pairs of complex conjugate numbers, then $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$ equals

(1) 0 (2) $\frac{\pi}{4}$
(3) $\frac{3\pi}{2}$ (4) $\pi$

**10.** Let $z_1$ and $z_2$ be two complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg(z_1) - \arg(z_2)$ equals

(1) 0 (2) $\frac{\pi}{4}$
(3) $\frac{3\pi}{2}$ (4) $\pi$

**11.** Let $z_1$ and $z_2$ be two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$ then

(1) $z_1 = z_2$ (2) $z_1 = -z_2$
(3) $z_1 = \bar{z}_2$ (4) $z_1 = -\bar{z}_2$

**12.** If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then

(1) $zw = 1$ (2) $zw = -i$
(3) $\bar{z}w = -i$ (4) None of these

**13.** If $|z_1| = |z_2| = 2$, then the value of $|1 - \bar{z}_1 z_2|^2 - |z_1 - z_2|^2$ equals

(1) 4 (2) 9
(3) 16 (4) 7

**14.** Let w ( Im w $\neq$ 0) be a complex number, then the set of all complex numbers z satisfying the equation $w - \bar{w}z = k(1 - z)$ for some real number k, is :

(1) $\{ z : |z| = 1 \}$ (2) $\{ z : z = \bar{z} \}$
(3) $\{ z = z \neq 1 \}$ (4) $\{ z : |z| = 1, z \neq 1 \}$

**15.** For all complex numbers z of the form $1 + i\alpha$, $\alpha \in \mathbb{R}$, if $z^2 = x + iy$, then

(1) $y^2 - 4x + 2 = 0$ (2) $y^2 + 4x - 4 = 0$
(3) $y^2 - 4x + 4 = 0$ (4) $y^2 + 4x + 2 = 0$

**16.** Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number, then $z+\frac{1}{z}$ is :

(1) 0
(2) any non-zero real number other than 1
(3) any non-zero real number
(4) a purely imaginary number

**17.** If $z_1$, $z_2$ are two complex numbers (not purely real) such that $Im(z_1 + z_2) = 0$ and $Im(z_1 z_2) = 0$, then

(1) $\bar{z}_1 = \bar{z}_2$
(2) $z_1 = z_2$
(3) $z_1 = \bar{z}_2$
(4) None of these

**18.** Number of solution of the equation $z^3 + \frac{3(\bar{z})^2}{|z|} = 0$ where z is a complex number is

(1) 2
(2) 3
(3) 6
(4) 5

**18.** If $z_1$, $z_2$, are the roots of the quadratic equation $az^2 + bz + c = 0$ such that $Im\ (z_1 z_2) \neq 0$ then

(1) a, b, c are all real
(2) at least one of a, b, c is real
(3) at least one of a, b, c is imaginary
(4) all of a, b, c are imaginary

**20.** The point of intersection the curves $\arg(z - i + 2) = \frac{\pi}{6}$ and $\arg\ (z + 4 - 3i) = -\frac{\pi}{4}$ is given by

(1) $(-2 + i)$
(2) $2 - i$
(3) $2 + i$
(4) none of these

**21.** The locus of z satisfying $|z - z_1| = |z - z_2|$, where $z_1$ and $z_2$ are fixed points, is

(1) line joining $z_1$ and $z_2$
(2) perpendicular bisector of line joining $z_1$ and $z_2$
(3) circle with end points of a diameter as $z_1$ and $z_2$
(4) No z satisfies the given equation

**22.** If $|\ z + 6\ | = |\ 2z + 3\ |$, then z lies on

(1) a straight line
(2) a parabola
(3) an ellipse
(4) a circle

**23.** $z = x + iy$, then $|3z - 1| = 3|z - 2|$ represents

(1) real axis
(2) imaginary axis
(3) a circle
(4) line parallel to the imaginary axis

**24.** If both the real and imaginary parts of z are positive, then

(1) $|\ z - 4\ | > |\ z - 2\ |$
(2) $|\ z - 4\ | < |\ z - 2\ |$
(3) $|\ z - 4\ | = |\ z - 2\ |$
(4) Cannot be determined

**25.** Locus of the point z satisfying Re $\left(\frac{1}{z}\right)$ = k, k is a non-zero real number, is

(1) a straight line
(2) a circle
(3) an ellipse
(4) a hyperbola

**26.** If $u = \frac{z}{z - \frac{1}{2}i}$ and |u| = 1, then z lies on:

(1) a straight line
(2) a parabola
(3) an ellipse
(4) a circle

**27.** If a complex number z lies in the interior or on the boundary of a circle of radius "3" units and centre (– 4, 0), then maximum value of |z +1 | equals

(1) 3
(2) 4
(3) 5
(4) 6

**28.** What is the maximum value of |z|, when z satisfies the condition $\left|z + \frac{3}{z}\right| = 4$?

(1) $\frac{13}{4}$
(2) 4
(3) $\frac{19}{4}$
(4) None of these

**29.** The locus of z if |z – 4| + |z + 2| = 4 is

(1) an ellipse
(2) a parabola
(3) a circle
(4) There is no z that satisfies the condition.

**30.** The points of z satisfying arg $\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ lies on

(1) an arc of a circle
(2) a parabola
(3) an ellipse
(4) a straight line

**31.** Let S denote the set of all complex numbers z satisfying the inequality $|z - 5i| \le 3$. The complex numbers z in S having least postive argument is :

(1) $\frac{12-16i}{5}$
(2) $\frac{16+12i}{5}$
(3) $\frac{16-12i}{5}$
(4) $\frac{12+16i}{5}$

**32.** If $|z-1|+|z+3|<8,$ the the range of values of $|z-4|$ is

(1) [0, 8] (2) [1, 8]

(3) [1, 9] (4) [– 3, 5]

**33.** The largest value of r for which the region represented by the set $\{\omega \in C / |\omega - 4 - i| \le r\}$ is contained in the region represented by the set $\{z \in C / |z-1| \le |z+i|\}$, is equal to :

(1) $\sqrt{17}$ (2) $2\sqrt{2}$

(3) $\frac{3}{2}\sqrt{2}$ (4) $\frac{5}{2}\sqrt{2}$

**34.** If |z| = 5, then the points representing the complex number $i+\frac{15}{z}$ lies on the circle

(1) whose centre is (0 , 1) and radius = 3 (2) whose centre is (0 ,– 1) and radius = 3

(3) whose centre is (1 , 0) and radius = 15 (4) whose centre is (–1 , 0) and radius = 15

**35.** If z is a non-zero real complex number, then the minimum value of $\frac{\text{Im}(z^5)}{(\text{Im } z)^5}$ is :

(1) – 1 (2) – 2

(3) – 4 (4) – 5

**36.** The product of all the three the cube roots of "–1" is equal to

(1) 0 (2) 1

(3) – 1 (4) None of these

**37.** If the cube roots of unity are 1, $\omega$, $\omega^2$, then the roots of the equation $(x - 1)^3 + 8 = 0$, are:

(1) $-1, 1-2\omega, 1-2\omega^2$ (2) $-1, 1+2\omega, 1+2\omega^2$

(3) $-1, -1+2\omega, -1-2\omega^2$ (4) – 1, – 1, – 1

**38.** If $\omega$ is a complex cube root of unity, then $(1-\omega+\omega^2)^6 + (1-\omega^2+\omega)^6 =$

(1) 0 (2) 6

(3) 64 (4) 128

**39**. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of

$$\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots+\left(z^6+\frac{1}{z^6}\right)^2 \text{ is}$$

(1) 12 (2) 18
(3) 54 (4) 6

**40.** Sum of common roots of the equations: $z^3+2z^2+2z+1=0$ and $z^{97}+z^{29}+1=0$ is equal to
(1) 0 (2) –1
(3) 1 (4) None of these

**41.** $\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\ldots+\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)=$

(1) $\frac{n\left(n^2+2\right)}{3}$ (2) $\frac{n\left(n^2+1\right)}{3}$

(3) $\frac{n(n+1)(n+2)}{3}$ (4) None of these

**42.** If 1, $\omega$, $\omega^2$ ... $\omega^{n-1}$ are the roots of the expression $x^n - 1$, then find the argument of $\omega^2$.

(1) $\frac{2\pi}{n}$ (2) $\frac{4\pi}{n}$

(3) $\frac{6\pi}{n}$ (4) $\frac{8\pi}{n}$

**43.** The value of $(1-\omega)(1-\omega^2)(1-\omega^3)\ldots(1-\omega^{n-1})$ if 1, $\omega$, $\omega^2$ ..., $\omega^{n-1}$ are the n roots of $x^n = 1$, is
(1) 0 (2) n
(3) n – 1 (d5 None of these

**44.** If 1, $\omega$, $\omega^2$, ..., $\omega^{n-1}$ are the nth roots of unity and n is an even number, then $(1+\omega)(1+\omega^2)\ldots(1+\omega^{n-1}) =$
(1) 1 (2) 0
(3) –1 (4) None of these

**45.** The value of $\sum_{k=1}^{6}\left[\sin\frac{2k\pi}{7}-i\cos\frac{2k\pi}{7}\right]$ is

(1) 0 (2) 1
(3) i (4) –i

**46**. The value of $\sum_{k=1}^{8}\left(\sin\frac{2k\pi}{9}+i\cos\frac{2k\pi}{9}\right)$ is

(1) 1 (2) –1
(3) i (4) – i

**47.** If a particle starts from point A ( –3 , 4) and move by $45^0$ in anticlockwise direction along a circular path with centre as origin and reaches point B. New coodinates of B :

(1) $\left(\frac{3}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ (2) $\left(-\frac{3}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

(3) $\left(-\frac{7}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (4) $\left(\frac{7}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

**48.** The area of the triangle formed by the points represented by z , iz and z +iz in an argand plane is

(1) $|z|^2$ (2) $\frac{1}{2}|z|^2$

(3) $\frac{1}{3}|z|^2$ (4) None of these

**49.** $z_1$ and $z_2$ are the two roots of the equation $z^2 + az + b = 0$, z being complex.
If the origin, $z_1$, $z_2$ form an equilateral triangle, then
(1) $a^2 = b$ (2) $a^2 = 2b$
(3) $a^2 = 3b$ (4) $a^2 = 4b$

**50.** If $z_1$, $z_2$ and $z_3$ are the vertices of an equilateral triangle with $z_0$ as its centroid, then $z_1^2 + z_2^2 + z_3^2 =$
(1) $z_0^2$ (2) $9z_0^2$
(3) $3z_0^2$ (4) $2z_0^2$

**51.** Let $A(z_1)$ , $B(z_2)$ and $C(z_3)$ are vertices of an isoceles right angled triangle at $C(z_3)$ then $\frac{(z_1-z_2)^2}{(z_1-z_3)(z_3-z_2)}$ equals
(1) 1 (2) 2
(3) 3 (4) 4

**52.** If $z_1^2 + z_2^2 \pm 2z_1z_2\cos\theta = 0$, where $\theta$ is real and $A(z_1)$, $B(z_2)$ and O(origin) are vertices of

(1) equilateral triangle (2) Isoceles triangle
(3) obtuse angled triangle (4) None of these

**53.** If diagonals of a square intersects at $z_0 = 1 + i$ and one of the vertices is at the point $z_1 = 1 - i$. One of the other vertices could be :

(1) 1 + 2i (2) 2 + i
(3) 3 + i (4) 3 – i

**2002**

**1.** z and w are two non zero complex no.s such that $|z| = |w|$ and $\arg z + \arg w = \pi$ then z equals

(1) $\overline{w}$ (2) $-\overline{w}$

(3) w (4) – w

**2.** If $|z-4| < |z-2|$, its solution is given by

(1) Re(z) > 0 (2) Re(z) < 0

(3) Re (z) > 3 (4) Re(z) > 2

**3.** The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally ($z$, $z_1$ and $z_2$ are complex numbers) will be

(1) an ellipse (2) a hyperbola

(3) a circle (4) none of these

**2003.**

**4.** If 1, $\omega$, $\omega^2$ are the cube roots of unity, then $D = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

(1) $\omega^2$ (2) 0

(3) 1 (4) $\omega$

**5.** If z and $\omega$ are two non-zero complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega =$

(1) – i (2) 1

(3) –1 (4) i

**6.** Let $z_1$ and $z_2$ be two roots of the equation $z^2 + az + b = 0$ being complex. Further, assume that the origin, $z_1$ and $z_2$ form an equilateral triangle. Then

(1) $a^2 = 4b$ (2) $a^2 = b$

(3) $a^2 = 2b$ (4) $a^2 = 3b$

**7.** If $\left(\frac{1+i}{1-i}\right)^x = 1$ then ( for n being a positive integer )

(1) x= 2n+1 (2) x = 4n

(3) x = 2n (4) x = 4n + 1

**2004**

**8.** Let z, w be complex numbers such that $\bar{z} + i\,\overline{w} = 0$ and $\arg(zw) = \pi$. Then arg (z) equals

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$

(3) $\frac{3\pi}{4}$ (4) $\frac{5\pi}{4}$

**9.** If z = x – iy and $z^{1/3} = p + iq$, then $\dfrac{\left(\dfrac{x}{p} + \dfrac{y}{q}\right)}{p^2 + q^2}$ is equal to

(1) 1 (2) –1
(3) 2 (4) – 2

**10.** If $|z^2 - 1| = |z|^2 + 1$, then z lies on
(1) real axis (2) imaginary axis
(3) a circle (4) an ellipse

**2005**

**11.** If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x - 1)^3 + 8 = 0$, are:

(1) $-1, 1 + 2\omega, 1 + 2\omega^2$ (2) $-1, 1 - 2\omega, 1 - 2\omega^2$

(3) –1, –1, –1 (4) $-1, -1 + 2\omega, -1 - 2\omega^2$

**12.** If $z_1$ and $z_2$ are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg $z_1$ – arg $z_2$ is equal to:

(1) $-\dfrac{\pi}{2}$ (2) 0

(3) $-\pi$ (4) $\dfrac{\pi}{2}$

**13.** If $w = \dfrac{z}{z - \dfrac{1}{3}i}$ and |w| = 1, then z lies on:

(1) a parabola (2) a straight line
(3) a circle (4) an ellipse

**2006**

**14.** The value of $\sum_{k=1}^{10}\left(\sin\dfrac{2k\pi}{11} + i\cos\dfrac{2k\pi}{11}\right)$ is

(1) – i (2) i
(3) 1 (4) – 1

**15.** If $z^2 + z + 1 = 0$, where z is a complex number, then the value of

$$\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \ldots + \left(z^6 + \frac{1}{z^6}\right)^2$$ is

(1) 12 (2) 18
(3) 54 (4) 6

**2007**

**16.** If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is

(1) 0 (2) 4
(3) 10 (4) 6

**2008**

**17.** The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is

(1) $\frac{1}{i-1}$ (2) $\frac{-1}{i-1}$

(3) $\frac{1}{i+1}$ (4) $\frac{-1}{i+1}$

**2009**

**18.** If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|Z|$ is equal to

(1) $\sqrt{3}+1$ (2) $\sqrt{5}+1$
(3) 2 (4) $2+\sqrt{2}$

**2010**

**19.** The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals

(1) $\infty$ (2) 0
(3) 1 (4) 2

**20.** If $\alpha$ and $\beta$ are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(1) 2 (2) –2
(3) –1 (4) 1

**2011**

**21.** Let $\alpha, \beta$ be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that

(1) $\beta \in (-1, 0)$ (2) $|\beta| = 1$

(3) $\beta \in (1, \infty)$ (4) $\beta \in (0, 1)$

**22.** If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals

(1) (1, 1) (2) (1, 0)
(3) (–1, 1) (4) (0, 1)

**2011 ( Test II )**

**23.** If $\omega \neq 1$ is the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then $H^{70}$ is equal to

(1) H (2) 0
(3) – H (4) $H^2$

**2012**

**24.** If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

(1) either on the real axis or on a circle not passing through the origin
(2) on the imaginary axis
(3) either on the real axis or on a circle passing through the origin
(4) on a circle with centre at the origin

**2013**

**25.** If "z" is a complex number of unit modulus and argument "$\theta$", then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals

(1) $-\theta$ (2) $\frac{\pi}{2}-\theta$

(3) $\theta$ (4) $\pi-\theta$

**2014**

**26.** If z is a complex number such that $|z| \geq 2$, then the mimimum value of $\left|z+\frac{1}{2}\right|$

(1) is strictly greater than 5/2 (2) is strictly greater than 3/2 but less than 5/2
(3) is equal to 5/2 (4) lies in the interval (1 , 2)

**2015**

**27.** A complex number z is said to be unimodular if |z| = 1. Suppose $z_1$ and $z_2$ are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\overline{z_2}}$ is unimodular and $z_2$ is not unimodular. Then the point $z_1$ lies on :

(1) straight Line parallel to y–axis (2) circle of radius 2
(3) circle of radius $\sqrt{2}$ (4) straight line parallel to x – axis

**2016**

**28.** A value of $\theta$ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$

(3) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

**1.** The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is

(1) 8 (2) 16
(3) 12 (4) 4

**2.** The complex number z = x + iy which satisfy the equation $\left|\frac{z-5i}{z+5i}\right| = 1$ , lie on

(1) the x-axis (2) the straight line y = 5
(3) a circle passing through the origin (4) None of the above

**3.** If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^3 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then

(1) Re (z) = 0 (2) Im(z) = 0
(3) Re(z) > 0 ; Im(z) > 0 (4) Re(z) > 0 ; Im(z) < 0

**4.** The inequality $|z-4| < |z-2|$ represents the region given by

(1) $Re(z) \geq 0$ (2) $Re(z) < 0$

(3) $Re(z) > 0$ (4) None of these

**5.** If z = x + iy and w = (1 – iz) / (z – i) then $|w| = 1$ implies that, in the complex plane

(1) z lies on the imaginary axis (2) z lies on the real axis
(3) z lies on the unit circle (4) None of above

**6.** The points $z_1$, $z_2$, $z_3$ , $z_4$ in the complex plane are the vertices of a parallelogram taken in order, if and only if

(1) $z_1 + z_4 = z_2 + z_3$ (2) $z_1 + z_3 = z_2 + z_4$
(3) $z_1 + z_2 = z_3 + z_4$ (4) None of these

**7.** If a, b, c and u, v, w are the complex numbers representing the vertices of two triangles such that c = (1 – r) a + rb and w = (1 – r) u + rv, where r is a complex number, then the two triangles

(1) have the same area (2) are similar
(3) are congruent (4) None of these

**8.** The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$ is

(1) – 1 (2) 0
(3) – i (4) i

**9.** If $z_1$ and $z_2$ are two non-zero complex numbers such that $|z_1+z_2|=|z_1|+|z_2|$, then arg $(z_1)-(z_2)$ is equal to

(1) $-\pi$ (2) $-\frac{\pi}{2}$

(3) 0 (4) $\frac{\pi}{2}$

**10.** The complex numbers sinx + icos2x and cosx – isin2x are conjugate to each other, for

(1) $x=n\pi$ (2) x= 0

(3) $x=(n+1/2)\pi$ (4) no value of x

**11.** If $\omega(\neq 1)$ is a cube root of unity and $(1+\omega)^7 = A+B\omega$, then A and B are respectively

(1) 0 , 1 (2) 1 , 1

(3) 1 , 0 (4) –1 , 1

**12.** Let z and w be two non-zero complex numbers such that $|z|=|w|$ and (z) + arg (w) = $\pi$, then z equals

(1) w (2) – w

(3) $\overline{w}$ (4) $-\overline{w}$

**13.** Let z and w be two complex numbers such that $|z|\le 1, |w|\le 1$ and $|z+iw|=|z-i\overline{w}|=2$, then z equals

(1) 1 or i (2) i or – 1

(3) 1 or – 1 (4) i or – 1

**14.** If $\omega$ is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ is equal to

(1) $128\omega$ (2) $-128\omega$

(3) $128\omega^2$ (4) $-128\omega^2$

**15.** The value of sum $\sum_{n=1}^{13}(i^n+i^{n+1})$, where $i=\sqrt{-1}$ equals

(1) i (2) i – 1

(3) – i (4) 0

**16.** If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x+iy$, then

(1) x = 3 , y = 1 (2) x = 1 , y =1

(3) x = 0 , y =3 (4) x = 0 , y = 0

**17.** $4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

(1) $1-i\sqrt{3}$ (2) $-1+i\sqrt{3}$

(3) $i\sqrt{3}$ (4) $-i\sqrt{3}$

**2000**

**18.** If arg (z) < 0, then arg (–z) – arg (z) equals

(1) $\pi$ (2) $-\pi$

(3) $-\pi/2$ (4) $\pi/2$

**2000**

**19.** If $z_1$, $z_2$ and $z_3$ are complex numbers such that $|z_1|=|z_2|=|z_3|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$, then $|z_1+z_2+z_3|$ is

(1) equal to 1 (2) less than 1

(3) greater than 3 (4) equal to 3

**2001**

**20.** Let $z_1$ and $z_2$ be nth roots of unity which subtend a right angled at the origin, then n must be of the form (where k is an integers)

(1) 4k + 1 (2) 4k + 2

(3) 4k + 3 (4) 4k

**2001**

**21.** The complex numbers $z_1$, $z_2$ and $z_3$ satisfying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$ are the vertices of a triangle which is

(1) of area zero (2) right-angled isosceles

(3) equilateral (4) obtuse-angled isosceles

**2002**

**22.** Let $\omega=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$, then value of the determinant $\begin{vmatrix}1 & 1 & 1\\ 1 & -1-\omega^2 & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}$ is

(1) $3\omega$ (2) $3\omega(\omega-1)$

(3) $3\omega^2$ (4) $3\omega(1-\omega)$

**23.** For all complex numbers $z_1$, $z_2$ satisfying $|z_1|=12$ and $|z_2-3-4i|=5$, the minimum value of $|z_1-z_2|$ is

(1) 0 (2) 2

(3) 7 (4) 17

**2003**

**24.** If $|z| = 1$ and $w = \frac{z-1}{z+1}$ (where z $\neq$ –1) , then Re (w) is

(1) 0
(2) $\frac{1}{|z+1|^2}$
(3) $\frac{1}{|z+1|}.\frac{1}{|z+1|^2}$
(4) $\frac{\sqrt{2}}{|z+1|^2}$

**2004**

**25.** If $\omega(\neq 1)$ be a cube root of unity and $\left(1+\omega^2\right)^n = \left(1+\omega^4\right)^n$, then the least positive value of n is

(1) 2
(2) 3
(3) 5
(4) 6

**2005**

**26.** The minimum value of $|a+b\omega+c\omega^2|$, where a, b and c are all not equal integers and $\omega(\neq 1)$ is a cube root of unity, is

(1) $\sqrt{3}$
(2) 1/2
(3) 1
(4) 0

**2005**

**27.** The region external to segment PQRS with P at centre , where $P = (-1,0)$, $Q = \left(-1+\sqrt{2}, \sqrt{2}\right)$
$R = \left(-1+\sqrt{2}, -\sqrt{2}\right), S = (1,0)$, is represented by

(1) $|z+1| > 2, |\arg(z+1)| < \frac{\pi}{4}$
(2) $|z+1| < 2, |\arg(z+1)| < \frac{\pi}{2}$
(3) $|z+1| > 2, |\arg(z+1)| > \frac{\pi}{4}$
(4) $|z+1| < 2, |\arg(z+1)| > \frac{\pi}{2}$

**2006**

**28.** If $w = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w - \overline{w}z}{1-z}\right)$ is purely real, then the set of values of z is

(1) $|z| = 1, z \neq 2$
(2) $|z| = 1$ and $z \neq 2$
(2) $z = \overline{z}$
(4) None of these

**2007**

**29.** A man walks a distance of 3 units from the origin towards the North-East (N 45° E) direction. From there, he walks a distance of 4 units towards the North-West (N 45° W) direction to each a point P. Then, the position of P in the Argand plane is

(1) $3e^{i\pi/4} + 4i$
(2) $(3-4i)e^{i\pi/4}$
(3) $(4+3i)e^{i\pi/4}$
(4) $(3+4i)e^{i\pi/4}$

**2007**

**30.** If | z | = 1 and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on

(1) a line not passing through the origin
(2) $|z| = \sqrt{2}$
(3) the x-axis
(4) the y-axis

**2008**

**31.** A particle P starts from the point $z_o = 1 + 2i$, where $i = \sqrt{-1}$. it moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point $z_1$. From $z_1$ the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point $z_2$. The point $z_2$ is given by

(1) 6 + 7i
(2) –7+6i
(3) 7+6i
(4) –6+7i

**2009**

**32.** Let $z = \cos\theta + i\sin\theta$. Then, the value of $\sum_{m=1}^{15} \text{Im}\left(z^{2m-1}\right)$ at $\theta = 2^o$ is

(1) $\frac{1}{\sin 2^o}$
(2) $\frac{1}{3\sin 2^o}$
(3) $\frac{1}{2\sin 2^o}$
(4) $\frac{1}{4\sin 2^o}$

**33.** Let z = x + iy be a complex number where x and y are integers. Then, the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is

(1) 48
(2) 32
(3) 40
(4) 80

**2011( Reformatted)**

**34.** If z is any complex number satisfying |z–3–2i| ≤ 2, then minimum value of |2z– 6 + 5i| is equal to

(1) 3
(2) 4
(3) 5
(4) 6

**2012**

**35.** Let z be a complex number such that the imaginary part of "z" is non zero and a = $z^2 + z + 1$ is real. Then "a" can not take the value

(1) –1
(2) 1/3
(3) 1/2
(4) 3/4

**2013**

**36.** Let complex numbers "a" and "1/a" lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then |a| =

(1) $\frac{1}{\sqrt{2}}$
(2) 1/2

(3) $\frac{1}{\sqrt{7}}$ (4) 1/3

**37.** Let w = $\frac{\sqrt{3}+i}{2}$ and $= \{ w^n : n = 1,2,3,...\}$. Further, $H_1 = \left\{ z \in C : \text{Re}(z) > \frac{1}{2} \right\}$ and $H_2 = \left\{ z \in C : \text{Re}(z) < -\frac{1}{2} \right\}$, where C is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1Oz_2$ equals

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{3}$ (4) $\frac{2\pi}{3}$

**2014**

**38.** Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$ ; k = 1,2,3,...9

| | **List I** | **List II** |
|---|---|---|
| **P.** | For each of $z_k$ there exists a $z_j$ such that $z_k . z_j = 1$ | 1. True |
| **Q.** | There exists a $k \in \{1,2,3,...9\}$ such that $z_1 . z = z_k$ has no solution z in the set of complex numbers | 2. False |
| **R.** | $\frac{\lvert 1-z_1\rvert \lvert 1-z_2\rvert ....\lvert 1-z_9\rvert}{10}$ | 3. 1 |
| **S.** | $1-\sum_{k=1}^{n-1}\cos\left(\frac{2k\pi}{10}\right)$ equals | 4. 2 |

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 1 | 2 | 4 | 3 |
| (B) | 2 | 1 | 3 | 4 |
| (C) | 1 | 2 | 3 | 4 |
| (D) | 2 | 1 | 4 | 3 |

**2015**

**39.** Let $\omega$ be complex cube root of unity and if $\left(3-3\omega+2\omega^2\right)^{4n+3} + \left(2+3\omega-3\omega^2\right)^{4n+3} + \left(-3+2\omega+3\omega^2\right)^{4n+3} = 0$, then no. of possible values of n in set A = { 1,2,3,4,5} is

(1) 1 (2) 2 (3) 3 (4) 4

## Questions for Practice @ JEE Main Level

| | | | | | |
|---|---|---|---|---|---|
| 1.(2) | 2.(3) | 3.(2) | 4.(1) | 5.(4) | 6.(3) |
| 7.(3) | 8.(4) | 9.(1) | 10.(1) | 11.(4) | 12.(3) |
| 13.(2) | 14.(4) | 15.(2) | 16.(3) | 17.(3) | 18.(4) |
| 19.(3) | 20.(4) | 21.(2) | 22.(4) | 23.(4) | 24.(4) |
| 25.(2) | 26.(1) | 27.(4) | 28.(4) | 29.(4) | 30.(1) |
| 31.(4) | 32.(3) | 33.(4) | 34.(1) | 35.(3) | 36.(3) |
| 37.(1) | 38.(4) | 39.(1) | 40.(2) | 41.(1) | 42.(2) |
| 43.(2) | 44.(2) | 45.(3) | 46.(4) | 47.(3) | 48.(2) |
| 49.(3) | 50.(3) | 51.(2) | 52.(2) | 53.(3) | |

## JEE Main / AIEEE Questions :

| | | | | | |
|---|---|---|---|---|---|
| 1.(2) | 2.(3) | 3.(2) | 4.(2) | 5.(1) | 6.(4) |
| 7.(2) | 8.(3) | 9.(4) | 10.(2) | 11.(2) | 12.(2) |
| 13.(2) | 14.(1) | 15.(1) | 16.(4) | 17.(4) | 18.(2) |
| 19.(3) | 20.(4) | 21.(3) | 22.(1) | 23.(1) | 24.(4) |
| 25.(3) | 26.(4) | 27.(2) | 28.(4) | | |

## JEE Advanced Single Option :

| | | | | | |
|---|---|---|---|---|---|
| 1.(4) | 2.(1) | 3.(2) | 4.(4) | 5.(2) | 6.(2) |
| 7.(2) | 8.(4) | 9.(3) | 10.(4) | 11.(2) | 12.(4) |
| 13.(3) | 14.(4) | 15.(2) | 16.(4) | 17.(3) | 18.(1) |
| 19.(1) | 20.(4) | 21.(3) | 22.(2) | 23.(2) | 24.(1) |
| 25.(2) | 26.(3) | 27.(1) | 28.(2) | 29.(4) | 30.(4) |
| 31.(4) | 32.(4) | 33.(1) | 34.(3) | 35.(4) | 36.(3) |
| 37.(4) | 38.(C) | 39.(4) | | | |